Finite Math - Spring 2017
Lecture Notes - 2/15/2017

Homework

- Section 3.2 - 9, 11, 13, 15, 17, 23, 33, 35, 43, 45, 61, 66, 76, 89

Section 3.1 - Simple Interest

Example 1. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59*,* \$19.95*, and* \$97.26 *are posted on days* 6*,* 13*, and* 25*, respectively, and a payment of* \$140 *is credited on day* 8*, what will be the balance on the card at the start of the next billing cycle?*

Solution. \$708.92

Section 3.2 - Compound and Continuous Compound Interest

Compound Interest. In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example 2. *Suppose* \$5,000 *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

Solution. *We find the future value at the end of the first quarter:*

$$A_1 = \$5,000\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,150.$$

This amount is carried into the second quarter and interest is computed again over the quarter:

$$A_2 = \$5,150\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,304.50.$$

We do this twice more to find a value at the end of the fourth quarter:

$$A_3 = \$5,304.50\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,463.635.$$

$$A_4 = \$5,463.635\left(1 + 0.12\left(\frac{1}{4}\right)\right) = \$5,627.54.$$

If we generalize this process, we end up with the following result

Definition 1 (Compound Interest)**.**

$$A = P(1+i)^n, \text{ where } i = \frac{r}{m}$$

The variables in this equation are

- $A =$ *future value after* n *compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $i =$ *rate per compounding period*
- $n =$ *total number of compounding periods*

Alternately, one can reinterpret this formula as a function of time as

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

where A, P, r, and m have the same meanings as above and t is the time in years.

Example 3. *If* $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution. *In this example, the quantities that will be changing are* m *and* n *(and thus also* i*). The fixed quantities are the principal* $P = \$1,000$ *and the annual nominal rate* $r = 0.06$*.*

(a) *Annually compounded means* $m = 1$*. Since we are going for* 8 *years, this means there will be* $n = 8(1) = 8$ *compounding periods. We also get* $i = \dfrac{0.06}{1} = 0.06$*, so the future value will be*

$$A = \$1,000(1+0.06)^8 = \$1,593.85.$$

(b) *Semiannually compounded means* $m = 2$*. Since we are going for* 8 *years, this means there will be* $n = 8(2) = 16$ *compounding periods. We also get* $i = \dfrac{0.06}{2} = 0.03$*, so the future value will be*

$$A = \$1,000(1+0.03)^{16} = \$1,604.71.$$

(c) *Quarterly compounded means* $m = 4$*. Since we are going for* 8 *years, this means there will be* $n = 8(4) = 32$ *compounding periods. We also get* $i = \dfrac{0.06}{4} = 0.015$*, so the future value will be*

$$A = \$1,000(1+0.015)^{32} = \$1,610.32.$$

(d) *Monthly compounded means* $m = 12$. *Since we are going for* 8 *years, this means there will be* $n = 8(12) = 96$ *compounding periods. We also get* $i = \dfrac{0.06}{12} = 0.005$, *so the future value will be*

$$A = \$1,000(1 + 0.005)^{96} = \$1,614.14.$$

Example 4. *If* \$2,000 *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after* 5 *years? How much interest is accrued in each case? Round answers to the nearest cent.*

Solution.

(a) \$2805.10 *with* \$805.10 *in interest.*

(b) \$2829.56 *with* \$829.56 *in interest.*

(c) \$2835.25 *with* \$835.25 *in interest.*

Continuous Compound Interest. Consider again the formulation of compound interest given by

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

$$\begin{aligned} A &= P\left(1 + \frac{r}{m}\right)^{mt} \\ &= P\left(1 + \frac{r}{m}\right)^{mt\cdot\frac{r}{r}} \\ &= P\left(1 + \frac{r}{m}\right)^{\left(\frac{m}{r}\right)rt} \\ &= P\left(1 + \frac{1}{x}\right)^{xrt} \qquad \left(x = \frac{m}{r}\right) \\ &= P\left[\left(1 + \frac{1}{x}\right)^{x}\right]^{rt} \end{aligned}$$

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

$$A = Pe^{rt}.$$

Definition 2 (Continuous Compound Interest)**.** *Principal* P *invested at an annual nominal rate* r *will have future value*

$$A = Pe^{rt}$$

after time t *(in years).*

Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.